



TRANSONIC WING REDESIGN USING A GENERALIZED FICTITIOUS GAS METHOD

N.J. Yu and P.E. Rubbert

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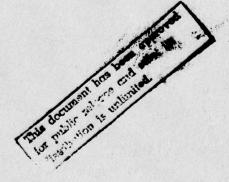
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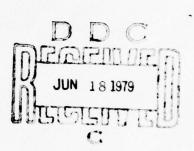
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Table of Contents

| I. Summary II. Introduction III. Formulation of Governing Equations IV. Redesign Procedure V. Numerical Results VII. Conclusion VII. Acknowledgement VIII. References List of Figures Figure 1. Real density (curve 0) versus fictitious density (curves 1 to 5) Figure 2a. Sonic surfaces generated by the original (curve 0) and the fictitious gas (curves 1 to 5) analysis for an advanced wing section at M _∞ = .75 and d = 10 Figure 2b. Original (curve 0) and redesigned shapes (curves 1 to 5) Figure 2c. Pressure distributions at design condition M _∞ = .75 and d = 10 Figure 2c. Off-design characteristics at M _∞ = .75 Figure 2c. Off-design pressure distributions for the original airfoil at M _∞ = .75 Figure 2f. Off-design pressure distributions for the redesigned airfoil-case 3 at M _∞ = .75 Figure 3a. Sonic surfaces of an advanced wing section at M _∞ = .75 | | | | Page No. |
|---|--------------------------|---|--|-----------------------------|
| List of Figures Figure 1. Real density (curve 0) versus fictitious density (curves 1 to 5) Figure 2a. Sonic surfaces generated by the original (curve 0) and the fictitious gas (curves 1 to 5) analysis for an advanced wing section at M _∞ = .75 and d = 10 Figure 2b. Original (curve 0) and redesigned shapes (curves 1 to 5) Figure 2c. Pressure distributions at design condition M _∞ = .75 and d = 10 Figure 2d. Off-design characteristics at M _∞ = .75 Figure 2e. Off-design pressure distributions for the original airfoil at M _∞ = .75 Figure 2f. Off-design pressure distributions for the redesigned airfoil-case 3 at M _∞ = .75 | II. III. IV. V. | Introduction Formulation of Governing Equations Redesign Procedure Numerical Results Conclusion | NTIS (I DDC TAB Unannusced Justification By Distributed Available Por | 1 1 3 4 8 11 |
| Figure 1. Real density (curve 0) versus fictitious density (curves 1 to 5) 16 Figure 2a. Sonic surfaces generated by the original (curve 0) and the fictitious gas (curves 1 to 5) analysis for an advanced wing section at $M_{\infty} = .75$ and $\omega = 10$ 17 Figure 2b. Original (curve 0) and redesigned shapes (curves 1 to 5) 18 Figure 2c. Pressure distributions at design condition $M_{\infty} = .75$ and $\omega = 10$ 19 Figure 2d. Off-design characteristics at $M_{\infty} = .75$ 20 Figure 2e. Off-design pressure distributions for the original airfoil at $M_{\infty} = .75$ 21 Figure 2f. Off-design pressure distributions for the redesigned airfoil-case 3 at $M_{\infty} = .75$ 22 | VIII. | References | a span al | 13 |
| Figure 2a. Sonic surfaces generated by the original (curve 0) and the fictitious gas (curves 1 to 5) analysis for an advanced wing section at $M_{\infty} = .75$ and $\mathcal{A} = 1^{\circ}$ 17 Figure 2b. Original (curve 0) and redesigned shapes (curves 1 to 5) 18 Figure 2c. Pressure distributions at design condition $M_{\infty} = .75$ and $\mathcal{A} = 1^{\circ}$ 19 Figure 2d. Off-design characteristics at $M_{\infty} = .75$ 20 Figure 2e. Off-design pressure distributions for the original airfoil at $M_{\infty} = .75$ 21 Figure 2f. Off-design pressure distributions for the redesigned airfoil-case 3 at $M_{\infty} = .75$ 22 | Figure 1. | | | |
| and the fictitious gas (curves 1 to 5) analysis for an advanced wing section at $M_{\infty} = .75$ and $\omega = 10$ 17 Figure 2b. Original (curve 0) and redesigned shapes (curves 1 to 5) 18 Figure 2c. Pressure distributions at design condition $M_{\infty} = .75$ and $\omega = 10$ 19 Figure 2d. Off-design characteristics at $M_{\infty} = .75$ 20 Figure 2e. Off-design pressure distributions for the original airfoil at $M_{\infty} = .75$ 21 Figure 2f. Off-design pressure distributions for the redesigned airfoil-case 3 at $M_{\infty} = .75$ 22 | | (curves 1 to 5) | | 16 |
| Figure 2b. Original (curve 0) and redesigned shapes (curves 1 to 5) 18 Figure 2c. Pressure distributions at design condition $M_{\infty} = .75$ and $\alpha = 10$ 19 Figure 2d. Off-design characteristics at $M_{\infty} = .75$ 20 Figure 2e. Off-design pressure distributions for the original airfoil at $M_{\infty} = .75$ 21 Figure 2f. Off-design pressure distributions for the redesigned airfoil-case 3 at $M_{\infty} = .75$ 22 | Figure 2a | | | |
| Figure 2c. Pressure distributions at design condition $M_{\infty} = .75$ and $\alpha = 10$ 19 Figure 2d. Off-design characteristics at $M_{\infty} = .75$ 20 Figure 2e. Off-design pressure distributions for the original airfoil at $M_{\infty} = .75$ 21 Figure 2f. Off-design pressure distributions for the redesigned airfoil-case 3 at $M_{\infty} = .75$ 22 | | an advanced wing section at M | ∞ = .75 and ω = 10 | 17 |
| Figure 2d. Off-design characteristics at $M_{\infty} = .75$ 20 Figure 2e. Off-design pressure distributions for the original airfoil at $M_{\infty} = .75$ 21 Figure 2f. Off-design pressure distributions for the redesigned airfoil-case 3 at $M_{\infty} = .75$ 22 | Figure 2b | . Original (curve 0) and redesig | gned shapes (curves 1 to 5 | 5) 18 |
| Figure 2d. Off-design characteristics at $M_{\infty} = .75$ 20 Figure 2e. Off-design pressure distributions for the original airfoil at $M_{\infty} = .75$ 21 Figure 2f. Off-design pressure distributions for the redesigned airfoil-case 3 at $M_{\infty} = .75$ 22 | Figure 2c | . Pressure distributions at des | ign condition $M_{\infty} = .75$ | |
| Figure 2e. Off-design pressure distributions for the original airfoil at $M_{\infty} = .75$ 21 Figure 2f. Off-design pressure distributions for the redesigned airfoil-case 3 at $M_{\infty} = .75$ 22 | | and $\alpha = 10$ | | 19 |
| airfoil at M_{∞} = .75 21 Figure 2f. Off-design pressure distributions for the redesigned airfoil-case 3 at M_{∞} = .75 22 | Figure 2d | . Off-design characteristics at | M _∞ = .75 | 20 |
| Figure 2f. Off-design pressure distributions for the redesigned airfoil-case 3 at M_{∞} = .75 | Figure 2e | . Off-design pressure distribut | ions for the original | |
| airfoil-case 3 at $M_{\infty} = .75$ | | airfoil at $M_{\infty} = .75$ | | 21 |
| | Figure 2f | | ions for the redesigned | |
| Figure 3a. Sonic surfaces of an advanced wing section at $M_{\infty} = .75$ | | | | |
| | Figure 3a | | | 5 |
| and | | | | 23 |

| Figure 3b. | Comparison of redesigned shapes | 24 |
|------------|---|----|
| Figure 4a. | Sonic surfaces generated by the original (curve 0) | |
| | and the fictitious gas analysis (curves 3, 4 and 5) | |
| | for a nonlifting rectangular wing at M $_{\infty}$ = .925 and | |
| | AR = 4. | 25 |
| Figure 4b. | Thickness reductions on the upper surface of the | |
| | redesigned wing at three span stations | 26 |
| Figure 4c. | Original (curve 0) and redesigned (curve 3, 4, and 5) | |
| | pressure distributions | 27 |

TRANSONIC WING REDESIGN USING A GENERALIZED FICTITIOUS GAS METHOD

I. SUMMARY

A numerical method for transonic shock-free or nearly shock-free airfoil and wing redesign based on the full potential equation is presented. The method utilizes a generalized fictitious gas approach wherein a variety of parameters controlling the character of the fictitious gas laws are introduced to provide a degree of control over the redesigned upper surface geometry and the pressure distribution of the redesigned shape. Results for a redesigned advanced airfoil as well as a three-dimensional wing are illustrated. Significantly improved aerodynamic characteristics are achieved through the present redesign technique.

II. INTRODUCTION

It is well known that the subsonic cruise speeds of high performance aircraft are limited by the onset of the transonic drag rise. Such drag rise is primarily caused by the formation of shock waves which recompresses the local supersonic flow to subsonic flow. One means for delaying the drag rise and its associated unfavorable flow phenomena is to improve (redesign) the wing contour shape so as to delay the formation of strong shock waves.

Several transonic wing design concepts have been explored over the years. The peaky airfoil concept first introduced by $Pearcey^1$ leads to the design

of stable shock-free transonic airfoils. However this peaky type of airfoil usually does not display acceptable off-design characteristics. Experimental results indicate that at lower Mach number the high pressure peak near the leading edge can trigger early boundary layer separation and cause high drag levels. On the other hand, a design with a roof-top type pressure distribution may be favorable to the boundary layer flow, but it may not be a very stable shock-free design.

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Current design efforts include the hodograph-based works of Nieuwland², Bauer, Garabedian, and Korn³, Boerstoel⁴ and Sobieczky⁵, and the use of optimization techniques (Hicks, Murman, and Vanderplaats⁶) for the design of supercritical airfoils without drag creep. Iterative finite difference methods have been explored by Steger and Klineberg⁷, Tranen⁸, Carlson⁹, Shankar, Malmuth, and $Cole^{10}$, and $Volpe^{12}$, wherein the design problem is treated as an inverse problem, i.e., the airfoil shape that supports a given pressure distribution is found by iteration. More recently, Sobieczky et al¹³., and Yu¹⁴ have developed a unique fictitious gas method for the design of shock-free airfoils and wings.

The present research extends the previous works¹³,¹⁴ to a general airfoil and wing redesign method. The method utilizes a generalized fictitious gas approach in the redesign process wherein a variety of parameters controlling the character of the fictitious gas laws are introduced to provide a degree of control over the redesigned upper surface geometry and the pressure distribution of the redesigned shape. The gas dependent redesigned configurations can be systematically varied to explore the trades between aerodynamic performance and geometry (thickness) changes in the transonic regime of flight.

III. FORMULATION OF GOVERNING EQUATIONS

The basic equations to be used in the present redesign procedure are the conservation of mass, energy and entropy for a potential flow, i.e.,

$$(P\phi_x)_x + (P\phi_y)_y + (P\phi_z)_z = 0,$$
 (1)

$$a^{2} + \frac{3-1}{2}g^{2} = \frac{1}{M_{\infty}^{2}} + \frac{3-1}{2}, \qquad (2)$$

$$\beta = \left(M_{\infty}^2 a^2 \right)^{\frac{1}{3^2 - 1}}, \tag{3}$$

respectively, where ϕ is the complete velocity potential, ρ , a, and q are the conventional definitions of the local density, speed of sound and flow speed respectively. All flow variables are normalized with respect to freestream conditions.

The present redesign technique requires modification of a reliable, fully conservative analysis code for the calculation of transonic flow over an initial input configuration. In two-dimensional redesign, one can either use Jameson's full potential ${\sf code}^{15}$ or Holst's approximate factorization (AF) ${\sf code}^{16}$ as a basis; where in three-dimensional redesign, we use a modified version of Jameson-Caughey's finite volume ${\sf code}^{17}$ for our redesign method.

IV. REDESIGN PROCEDURE

The present redesign method begins with the geometry of an initial input configuration that typically produces a shock wave terminating the upper surface supersonic bubble. The method then provides a local modification of the upper surface geometry underneath the supersonic bubble so as to produce a shock-free or nearly shock-free flow.

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The principal redesign concept embodies a temporary modification of the governing equations locally in the supersonic region so as to retain an elliptic behavior within the supersonic bubble-hence, the terminology "fictitious gas". In two dimensional flow, it can easily be proved that the following density laws will ensure elliptic behavior of the governing equations in the supersonic region. One, termed a continuous density law, is

$$\frac{P}{P^{\bullet}} = \left(1 + \frac{P_{i} C_{i}}{P_{i} + 1}\right) \left(\frac{8}{\alpha^{\bullet}}\right)^{-1} + C_{i} \left\{\frac{1}{P_{i} + 1} \left(\frac{2}{\alpha^{\bullet}}\right)^{P_{i}} - 1\right\}, \tag{4}$$

with both P_1 and C_1 greater than zero. Another is termed a discontinuous density law, meaning that the first derivative dP/dq is discontinuous at the sonic surface, and is given by

$$\frac{P}{P^*} = C_2 + (1 - C_2) \left(\frac{2}{a^*}\right)^{\frac{p_2}{2}}, \tag{5}$$

with $P_2 > -1$, $C_2 \geqslant 0$. In both equations the superscript "*" refers to the sonic condition. Figure 1 illustrates the density variations versus q/a* for several P_1 , C_1 and P_2 , C_2 .

In three-dimensional flow, only the discontinuous density law, equation (5), with proper combination of P_2 and C_2 has been used. More general density laws remain to be explored.

The governing equations and the boundary conditions in the subsonic regions remain unchanged. Numerical solutions to these modified, elliptic type governing equations contain no shock-wave-like discontinuities, and are correct in the subsonic regions up to the sonic boundaries that separate the subsonic and supersonic regions. The results within the supersonic bubble where the gas is "fictitious" are discarded.

The results along the sonic surface produced by the fictitious gas calculations are used as initial data for the recalculation of the flow field in the supersonic region using the real density law. In the two-dimensional airfoil redesign procedure, both a characteristics method and a marching method have been used, while in the three-dimensional study, only the marching method is used to recompute the continuous supersonic flow.

The characteristics method used in the two-dimensional supersonic flow calculation has been detailed in reference 13 and will only be briefly discussed here. Basically, the calculation is done in a hodograph-like working plane, in which the characteristics are orthogonal straight lines. The velocity potential ϕ and the stream function ψ are solved along the characteristics net, while the physical coordinates of each characteristic line are obtained from a simple integral relationship. Finally, the new redesigned airfoil geometry underneath the supersonic bubble that provides the continuous supersonic flow is obtained through interpolation of zero stream function in the flow field. Because the mass flux is exactly conserved in the

difference approximation, the redesigned geometry that produces a shock-free solution is smooth and continuous to the accuracy of the finite difference approximations.

The marching procedure which is used both in the two-dimensional airfoil and in the three-dimensional wing redesign is, in general, less accurate in numerical resolution, but is more flexible and easier to implement. Because the sonic surface data generated from the fictitious gas analysis are smooth and continuous, one can obtain the first derivatives of the flow variables on the surfaces, i.e., U_x , V_x , W_x , U_y , V_y , W_y and U_z , V_z , W_z through the use of Taylor series expansion of U, V, W along two arbitrary directions on the sonic surface, plus the continuity equation in quasilinear form

$$(\alpha^{2} - u^{2}) \mathcal{U}_{x} + (\alpha^{2} - v^{2}) \mathcal{V}_{y} + (\alpha^{2} - w^{2}) \mathcal{W}_{z}$$

$$-2 u v \mathcal{U}_{y} - 2 v w \mathcal{V}_{z} - 2 u w \mathcal{U}_{z} = 0,$$
(6)

and the three irrotationality conditions

$$\mathcal{U}_{\chi} = \mathcal{V}_{\mathcal{Y}}, \tag{7a}$$

$$\mathcal{U}_{\mathcal{Z}} = \mathcal{W}_{\mathcal{Z}}, \tag{7b}$$

$$v_z = iW_y. \tag{7c}$$

Once the derivative terms are calculated, one can then compute U, V and W on the next layer of computational surface through a direct numerical integration procedure. Here, as an illustrative example, we choose the coordinate Z - the direction normal to

the wing surface - as our marching direction. The flow variable on the next layer of computational surface is obtained by the trapezoidal integration formula

$$\mathcal{U}_2 = \mathcal{U}_1 + 0.5 \left(\mathcal{U}_{Z_1} + \mathcal{U}_{Z_2} \right), \tag{8}$$

here, subscript "1" and "2" refer to the old and new computational surfaces respectively. The marching procedure continues until the whole supersonic region is recomputed. The new wing shape and the pressure distribution associated with it are then extracted from the flow field solution. Here, the stream surface condition

$$\mathcal{U}\,\mathcal{Z}_{x}\,-\mathcal{W}\,+\,\mathcal{V}\,\mathcal{Z}_{y}\,=\,0,\tag{9}$$

is used to iterate the new wing surface coordinates. A more detailed formulation for this marching technique can be found in reference 14.

The final step of our redesign procedure consists in verifying the redesigned configuration to ensure that it does indeed provide a shock-free or a nearly shock-free pressure distribution. In two-dimensional airfoil redesign, the off-design characteristics of the resultant airfoils have also been studied.

V. NUMERICAL RESULTS

Two-Dimensional Airfoil Redesign

An advanced wing section at freestream conditions $M_{\infty} = .75$, d = 10 is selected as an illustrative example for the present redesign technique. Figures 2a-c show the sonic line shapes, the surface geometries and the pressure distributions of the original (curve 0) and the redesigned (curves 1 to 5) airfoils. Here curves 1 to 5 correspond to the redesigned results of the following density formulations in the supersonic region:

Table 1. Density laws used in the redesign procedure

| Curve | density equation | P ₁ or P ₂ | C ₁ or C ₂ |
|-------|------------------|----------------------------------|----------------------------------|
| 1 | 4 | 1 | 0.5 |
| 2 | 4 | 8 | 0.5 |
| 3 | 5 | -0.5 | 0 |
| 4 | 5 | 0.0 | 0 |
| 5 | 5 | 1.0 | 0 |

The density laws employed in the fictitious gas analysis exibit great control over the shape and the size of the sonic surface, which in turn, provide a variety of choices for the redesigned airfoil shape as well as its associated pressure distribution.

The off-design characteristics for the redesigned airfoils are plotted in Figure 2d, where the lift and drag coefficients are calculated at a fixed Mach number. Notice that there is a systematic correlation between the decreasing thickness of the airfoils and the inviscid drag improvements. Figures 2e and 2f illustrate the off-design pressure distributions for the original airfoil and the redesigned airfoil case 3. The redesigned shape produces a pressure distribution with weaker shock, and consequently, lower wave drag value for a range of lift coefficients. The effect of the redesigned pressure field on the boundary layer flow has not been investigated although this can easily be done by a simple interaction with a boundary layer code. For practical airfoil redesign, such interaction calculations would be important.

The present redesign procedure can also be readily implemented in other fully conservative analysis codes. To demonstrate that, we modified Holst's approximate factorization (AF) code for the fictitious gas analysis and repeated the redesign of the same airfoil using the constant density law $P = P^*$ in the supersonic region. The sonic surface shape and the redesigned airfoil geometry obtained from this code are compared with that of Jameson's full potential code in Figures 3a and 3b. The two results are practically identical.

8

The present two-dimensional redesign procedure is extremely efficient. The complete redesign process requires only one third of the CPU time of a corresponding transonic flow analysis calculation. This is not suprising, since the elliptic type of governing equation can be effectively solved by a fast direct solver 15.

Three-Dimensional Wing Redesign

The three dimensional redesign results illustrated here provide a demonstration of feasibility of the present redesign technique. Figures 4a-c illustrate the original and redesigned results for a nonlifting aspect ratio 4, rectangular wing with an NACA 64A series airfoil at $\rm M_{\infty}=.925$. The original thickness ratio tapered linearly from 6% chord at the root to .6% chord at the tip so as to generate a closed sonic surface inboard of the tip. The original configuration showed a rather strong shock at the inboard station, an expected phenomenon for a conventional NACA 6-digit wing section at a high subsonic speed. The redesigned results using the density laws 3, 4 and 5 in Table 1 are plotted on the same figures. Again, the use of different density laws in the redesign procedure effectively controls the redesigned shapes and their associated pressure distributions, although their effects are slightly less pronounced than in the two-dimensional studies. Both the shock wave and the wave drag have been effectively eliminated by the present redesign procedure.

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The computational efficiency for the three-dimensional redesign procedure is less effective than that of the two-dimensional redesign procedure. This is simply because we have not utilized the elliptic property of the modified governing equation. Currently, the three-dimensional redesign calculation requires about the same amount of CPU time as that of a typical transonic analysis calculation. Improvement on the computational efficiency utilizing accelerating schemes, such as Holst and Ballhaus' AF method 18, would be desirable for the further exploration of the present redesign method.

The present redesign method does not guarantee the existence of a shockfree solution for an arbitrary initial input configuration. In fact, both in two-dimensional and in three-dimensional redesign calculations, limit lines may occur for cases with a large and steep supersonic region. In such a case, the initial data along the sonic surface may be overspecified, and a continuous shock-free flow could not be obtained. Practically, this does not cause any serious problem, since an approximate solution which produces weak shock can usually be constructed from the present redesign method.

VI. CONCLUSION

A unique and practical transonic shock-free or nearly shock-free configuration redesign method has been explored. The method utilizes generalized fictitious gas laws to control the shape and the size of the supersonic region of a given initial configuration. The gas-dependent sonic surface determines the character of the redesigned configuration. The present method is especially effective for airfoil or wing redesign which is restricted to a minor modification of the upper surface geometry to achieve a favorable transonic pressure field. The method might also be particularly advantageous for variable geometry (adaptive) wing design work where it is desirable to obtain shock-free or nearly shock-free flow over a range of flight conditions by variation of the wing surface geometry. With some additional programming effort, the method can be extended to the redesign of wing-body combinations.

VII. ACKNOWLEDGEMENT

Part of the computing cost was supported by NASA-Ames Research Center under the sponsorship of Dr. W. F. Ballhaus. The author would like to thank him for his help and Dr. T. L. Holst for providing the 2-D analysis code.

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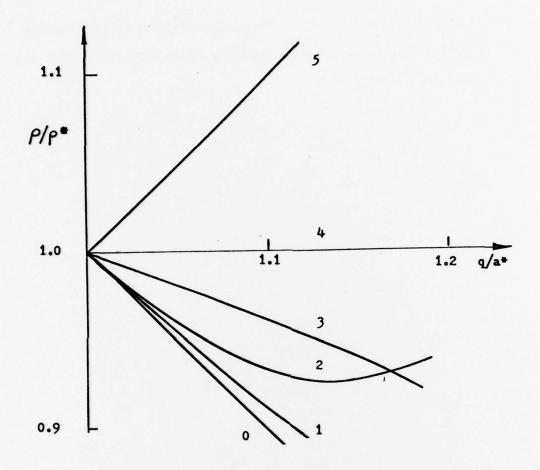
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| Curve | density equation | P ₁ or P ₂ | C ₁ or C ₂ |
|-------|------------------|----------------------------------|----------------------------------|
| 1 | 4 | 1 | 0.5 |
| 2 | 4 | 8 | 0.5 |
| 3 | 5 | -0.5 | 0 |
| 4 | 5 | 0.0 | 0 |
| 5 | 5 | 1.0 | 0 |

Figure 1. Real density (curve 0) versus fictitious density (curves 1 to 5)

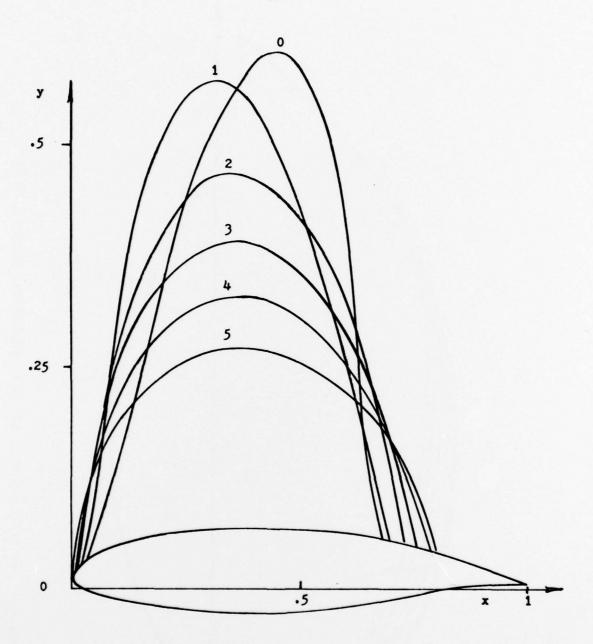
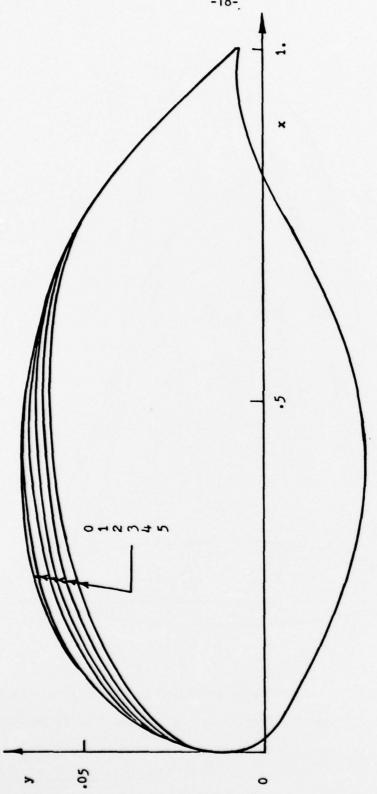
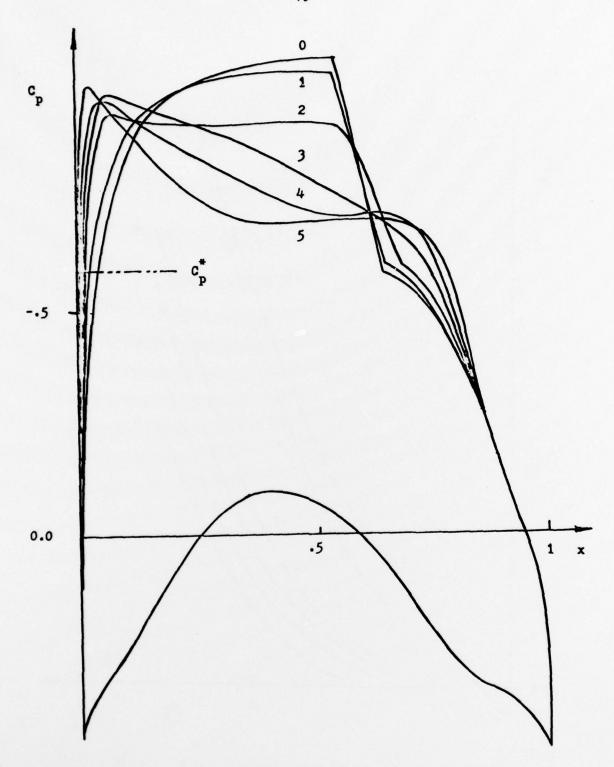


Figure 2a. Sonic surfaces generated by the original (curve 0) and the fictitious gas (curves 1 to 5) analysis for an advanced wing section at $M_{\infty}=.75$ and $\alpha = 10$



Original (curve 0) and redesigned shapes (curves 1 to 5) Figure 2b.



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Figure 2c. Pressure distributions at design condition $M_{\infty} = .75$ and d = 10

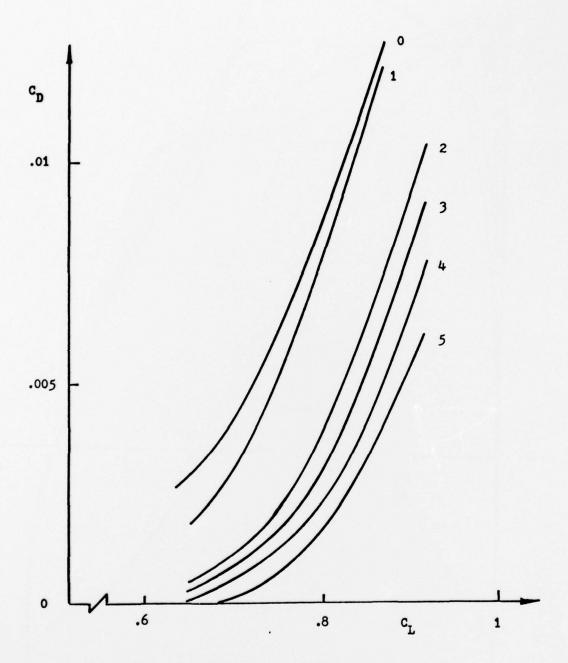


Figure 2d. Off-design characteristics at M_{∞} = .75

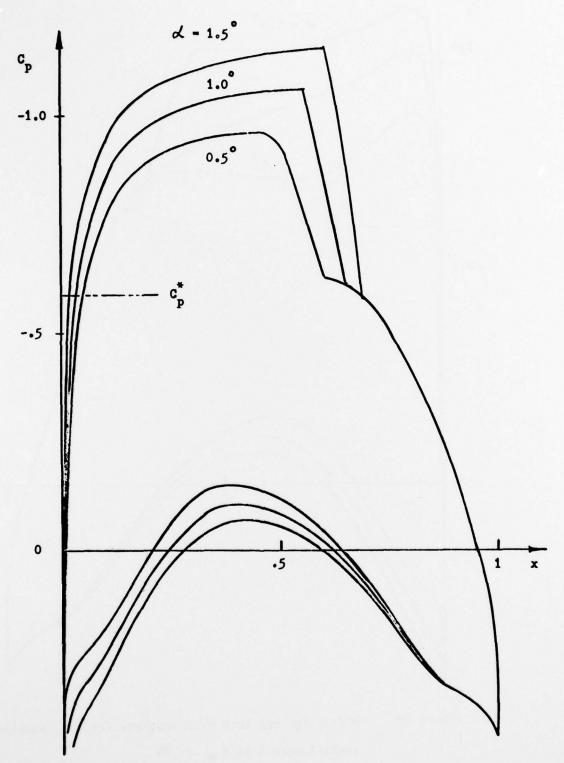


Figure 2e. Off-design pressure distributions for the original airfoil at M $_{\infty}$ = .75

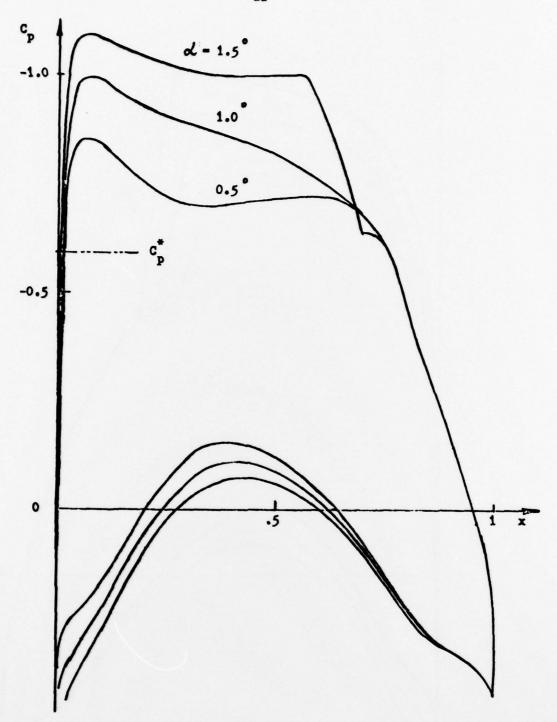


Figure 2f. Off-design pressure distributions for the redesigned airfoil-case 3 at M_{∞} = .75

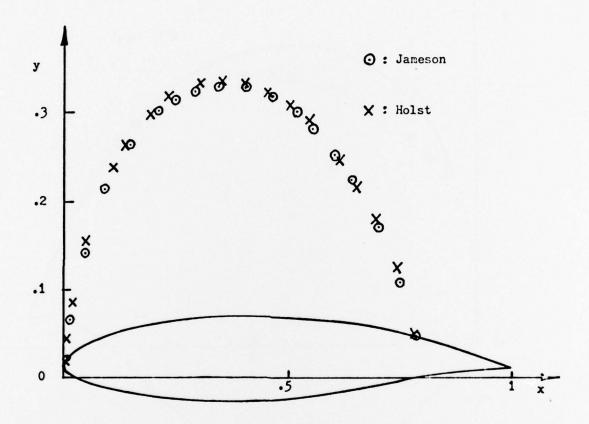


Figure 3a. Sonic surfaces of an advanced wing section at M $_{\infty}$ = .75 and α = 10 generated by the modified Jameson's and Holst's codes using constant density law - curve 4.

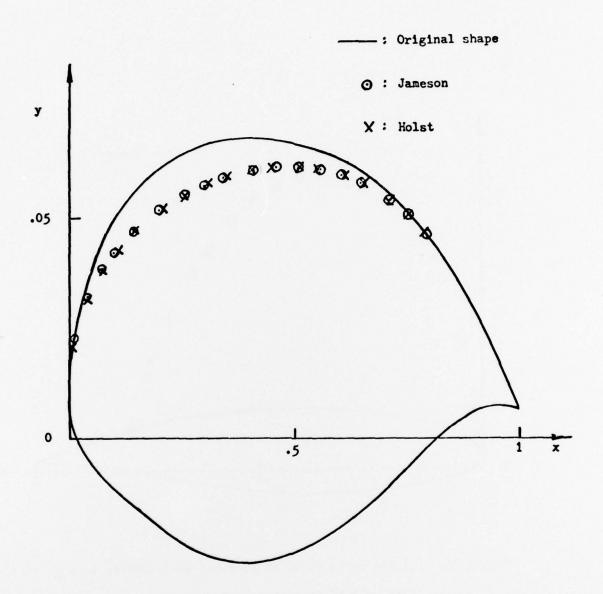
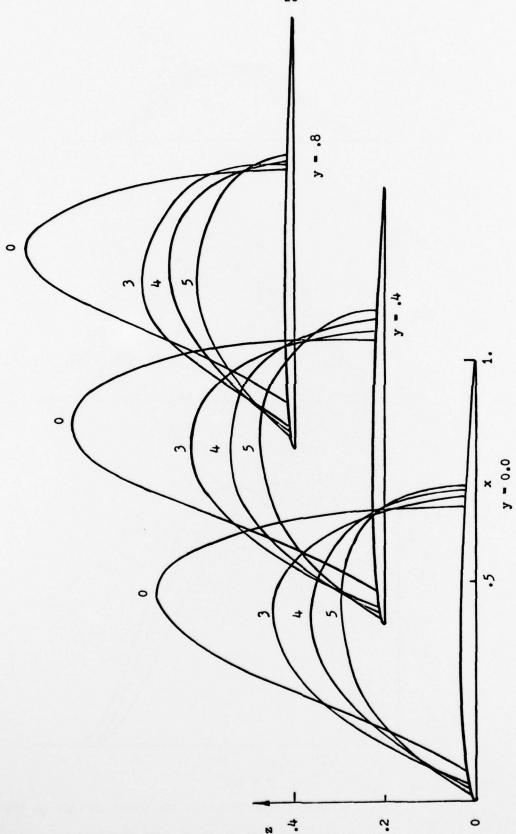
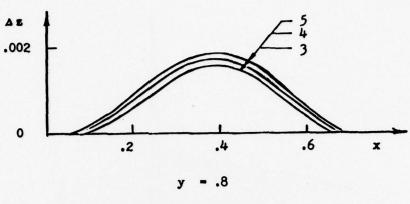


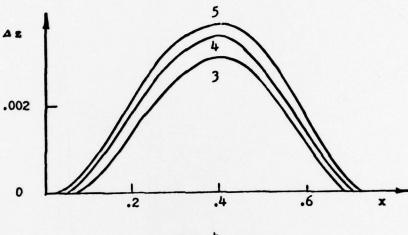
Figure 3b. Comparison of redesigned shapes



Sonic surfaces generated by the original (curve 0) and the fictitious gas analysis (curves 3, 4 and 5) for a nonlifting rectangular wing at M $_{\infty}$ = .925 and AR = 4. Figure 4a.







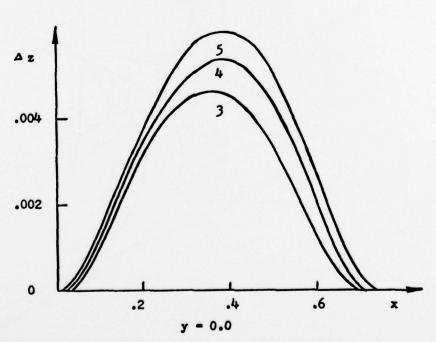


Figure 4b. Thickness reductions on the upper surface of the redesigned wing at three span stations

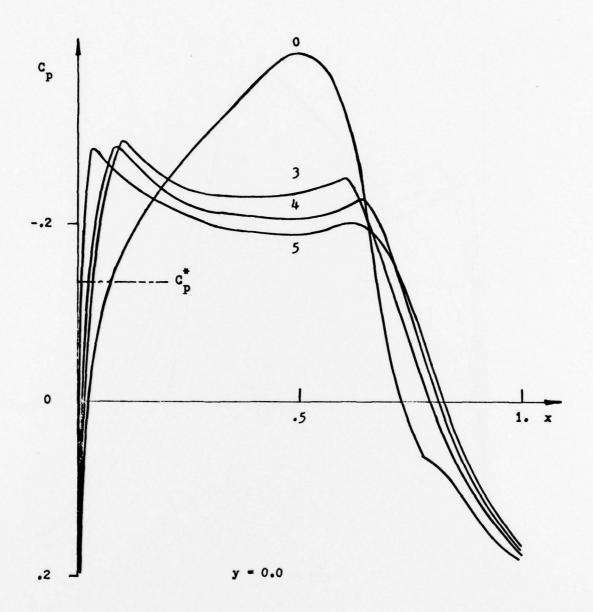


Figure 4c. Original (curve 0) and redesigned (curve 3, 4, and 5) pressure distributions

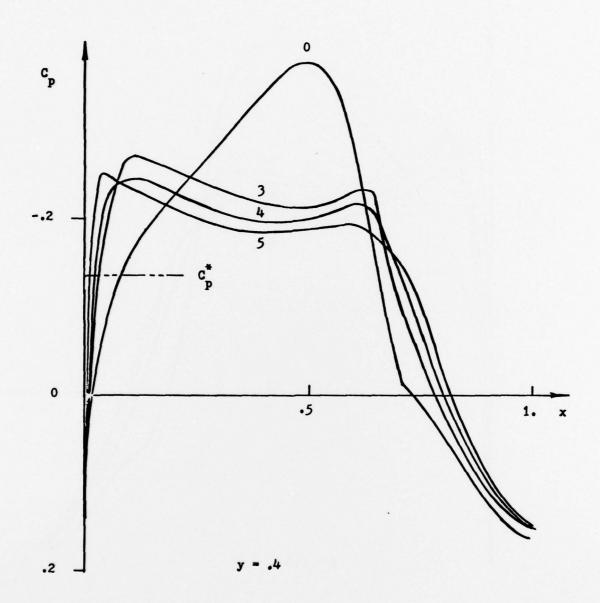


Figure 4c. Continued

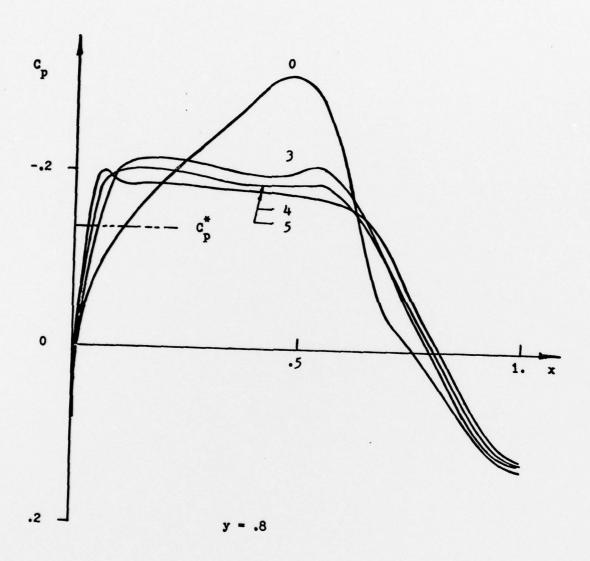


Figure 4c. Continued